$f(T)$ gravity and local Lorentz invariance

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We show that in theories of generalised teleparallel gravity, whose Lagrangians are algebraic functions of the usual teleparallel Lagrangian, the action and the field equations are *not* invariant under local Lorentz transformations. We also argue that these theories appear to have extra degrees of freedom with respect to general relativity. The usual teleparallel Lagrangian, which has been extensively studied and leads to a theory dynamically equivalent to general relativity, is an exception. Both of these facts appear to have been overlooked in the recent literature on $f(T)$ gravity, but are crucial for assessing the viability of these theories as alternative explanations for the acceleration of the universe.

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Teleparallel gravity [1, 2] is a gravity theory which uses the curvature-free Weitzenbock connection [3] to define the covariant derivative, instead of the conventional torsionless Levi-Civita connection of general relativity, and attempts to describe the effects of gravitation in terms of torsion instead of curvature. In its simplest form it is equivalent to general relativity (GR) but has a different physical interpretation [2]. Motivated by attempts to explain the observed acceleration of the universe in a natural way, there has been a great deal of recent interest in a generalisation of this theory in which the Lagrangian is an arbitrary algebraic function f of the Lagrangian of teleparallel gravity T. This is in direct analogy to creating $f(R)$ gravity theories as a generalisation of GR (see Ref. [16] for a review). This, so-called $f(T)$ gravity theory, has cosmological solutions which could provide alternative explanations for the acceleration of the universe [4–15]. The field equations for the $f(T)$ gravity have been claimed to be very different from those for $f(R)$ gravity, as they are second order rather than fourth order. This has been considered as an indication that the theory may be the more interesting relative of GR.

Here we will look further into the symmetries and dynamics of $f(T)$ gravity. Our main findings will be that such theories are not locally Lorentz invariant and appear to harbour extra degrees of freedom not present in GR. Remarkably, both of these features have been overlooked in the literature.

Let us briefly introduce teleparallel gravity and its $f(T)$ generalisation. Our dynamical variables are the vierbein or tetrad fields, $\mathbf{h}_a(x^\mu)$, which form an orthonormal basis for the tangent space at each point of the manifold with spacetime coordinates x^μ. Latin indices label tangent space coordinates while Greek indices label spacetime coordinates. All indices run from 0 to 3. Clearly $\mathbf{h}_a(x^\mu)$ is a vector in tangent space, and can be described in a coordinate basis by its components h^μ_a. So, h^μ_a is also a vector in spacetime.

The spacetime metric, $g_{\mu\nu}$, is given by

$$g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu\,, \tag{1}$$

where $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric for the tangent space. It follows that

$$h^\mu_a h^a_\nu = \delta^\mu_\nu, \quad h^\mu_a h^b_\mu = \delta^b_a, \tag{2}$$

where Einstein's summation convention has been used. GR uses the Levi-Civita connection

$$\Gamma^\lambda_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\sigma}\left(g_{\sigma\mu,\nu} + g_{\sigma\nu,\mu} - g_{\mu\nu,\sigma}\right), \tag{3}$$

in which commas denotes partial derivatives. This leads to nonzero spacetime curvature but zero torsion. In contrast, teleparallel gravity uses the Weitzenbock connection $\tilde{\Gamma}^\lambda{}_{\mu\nu}$(tilded to distinguish it from $\Gamma^\lambda_{\mu\nu}$),

$$\tilde{\Gamma}^\lambda{}_{\mu\nu} \equiv h^\lambda_b \partial_\nu h^b_\mu = -h^b_\mu \partial_\nu h^\lambda_b \tag{4}$$

which leads to zero curvature but nonzero torsion. The torsion tensor reads

$$T^\lambda{}_{\mu\nu} \equiv \tilde{\Gamma}^\lambda{}_{\nu\mu} - \tilde{\Gamma}^\lambda{}_{\mu\nu} = h^\lambda_b\left(\partial_\mu h^b_\nu - \partial_\nu h^b_\mu\right). \tag{5}$$

The difference between the Levi-Civita and Weitzenbock connections, which are not tensors, is a spacetime tensor, and is known as the contorsion tensor:

$$\begin{aligned} K^\rho{}_{\mu\nu} \equiv \tilde{\Gamma}^\rho{}_{\mu\nu} - \Gamma^\rho_{\mu\nu} &= \frac{1}{2}\left(T_\mu{}^\rho{}_\nu + T_\nu{}^\rho{}_\mu - T^\rho{}_{\mu\nu}\right) \\ &= h^\rho_a \nabla_\nu h^a_\mu, \end{aligned} \tag{6}$$

where ∇_ν denotes the metric covariant derivative.

If one further defines the tensor $S^{\rho\mu\nu}$ as

$$S^{\rho\mu\nu} \equiv K^{\mu\nu\rho} - g^{\rho\nu} T^{\sigma\mu}{}_\sigma + g^{\rho\mu} T^{\sigma\nu}{}_\sigma, \tag{7}$$

then the teleparallel lagrangian density is given by

$$\mathcal{L}_T \equiv \frac{h}{16\pi G} T \equiv \frac{h}{32\pi G} S^{\rho\mu\nu} T_{\rho\mu\nu}, \tag{8}$$

in which $h = \sqrt{-g}$ is the determinant of h^λ_a and g is the determinant of the metric $g_{\mu\nu}$, G is the gravitational constant and

$$\begin{aligned} T &\equiv \frac{1}{2} S^{\rho\mu\nu} T_{\rho\mu\nu} = -S^{\rho\mu\nu} K_{\rho\mu\nu} \\ &= \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu. \end{aligned} \tag{9}$$

Variation with respect to the tetrad h^a_λ after adding a matter Lagrangian density $\mathcal{L}_m$ leads to the field equations

$$h^{-1}\partial_\sigma\left(hh^\rho_a S_\rho{}^{\lambda\sigma}\right) - h^\sigma_a S^{\mu\nu\lambda}T_{\mu\nu\sigma} + \frac{1}{2}h^\lambda_a T = 8\pi G\,\Theta^\lambda_a, \tag{10}$$

where $\Theta^\lambda_a \equiv h^{-1}\delta\mathcal{L}_m/\delta h^a{}_\lambda$. The usual stress-energy tensor is given in terms of Θ^λ_a as $\Theta^{\mu\nu} = \eta^{ab}\Theta^\nu_a h^\mu_b$.

The $f(T)$ gravity theory generalises T in the lagrangian density to an arbitrary function of T:

$$\mathcal{L}_T \to \mathcal{L} \;=\; \frac{h}{16\pi G}f(T). \tag{11}$$

The derivation of field equations is very similar to that described above for teleparallel gravity. They are

$$\begin{aligned} f_T\left[h^{-1}\partial_\sigma\left(hh^\rho_a S_\rho{}^{\lambda\sigma}\right) - h^\sigma_a S^{\mu\nu\lambda}T_{\mu\nu\sigma}\right] \\ +f_{TT}h^\rho_a S_\rho{}^{\lambda\sigma}\partial_\xi T + \frac{1}{2}h^\lambda_a f(T) = 8\pi G\,\Theta^\lambda_a\,, \end{aligned} \tag{12}$$

where $f_T \equiv \partial f(T)/\partial T$ and $f_{TT} \equiv \partial^2 f(T)/\partial T^2$. Clearly, for $f(T) = T$, Eq. (12) reduces to Eq. (10).

We now move on to consider the symmetries of the action and the dynamical content of the field equations. When working in terms of tetrads and making explicit reference to a tangent space, two invariance principles should hold [17]: the action should be a generally covariant scalar, and so invariant under the infinitesimal coordinate transformations $x^\mu \to x^\mu + \epsilon^\mu(x)$; and if special relativity is to be recovered in locally inertial frames, the action must also be invariant under local (position-dependent) Lorentz transformations (*i.e.* we should be able to redefine the locally inertial coordinate systems at each point). Let us check if these properties hold for $f(T)$ gravity.

We start with the matter action, which in the literature is assumed to couple to the tetrad so as to couple effectively only to the metric. In this case the matter action is, as usual, both a generally covariant scalar and a Lorentz scalar[1]. It is worth considering the consequences of these assumptions for the matter action as an explicit example.

We denote an infinitesimal Lorentz transformation as $\Lambda^a{}_b(x^\mu) = \delta^a{}_b + \omega^a{}_b(x^\mu)$ with $|\omega^a{}_b| \ll 1$ and $\omega_{ab} = \omega_{[ab]}$. Square brackets denote anti-symmetrisation and parentheses symmetrisation. As the vierbein h^μ_a is a Lorentz vector in index a, it changes by $\delta h^\mu_a = \omega_a{}^b h^\mu_b$ under this Lorentz transformation, where we have suppressed the dependence on x^μ for simplicity. The matter action

$$\mathcal{S}_m = \int d^4x\mathcal{L}_m \tag{13}$$

is then changed by [2, 17]

$$\delta\mathcal{S}_m = \int \Theta^a{}_\mu h\delta h_a{}^\mu d^4x \;=\; \eta^{bc}\int \Theta^a{}_\mu h\omega_{ab}h_c{}^\mu d^4x\,. \tag{14}$$

ω_{ab} is an arbitrary antisymmetric (Lorentz) tensor, and

$$\eta^{bc}\Theta^a{}_\mu h_c{}^\mu \;=\; \eta^{ac}\Theta^b{}_\mu h_c{}^\mu \;\Leftrightarrow\; \Theta^{\beta\alpha} \;=\; \Theta^{\alpha\beta}, \tag{15}$$

so we see that $\delta\mathcal{S}_m = 0$ yields

$$\Theta^{\beta\alpha} = \Theta^{\alpha\beta}. \tag{16}$$

In other words, if $\mathcal{S}_m$ is invariant under local Lorentz transformations, then $\Theta_{\mu\nu}$ is symmetric, and vice versa.

Consider now the fact that the matter action is invariant under the infinitesimal coordinate transformation $x^\mu \to x^\mu + \epsilon^\mu(x)$ where $|\epsilon^\mu| \ll 1$. Under this transformation the vierbein changes by $\delta h^\mu_a(x) = h^\nu_a \epsilon^\mu_{,\nu} - h^\mu_{a,\lambda}\epsilon^\lambda$ [17] and the invariance of $\mathcal{S}_m$ yields

$$0 = \int d^4x\epsilon^\lambda\left[\partial_\nu\left(h\Theta^a{}_\lambda h^\nu_a\right) + h\Theta^a{}_\mu h^\mu_{a,\lambda}\right] \tag{17}$$

where we have dropped a total derivative. Now ϵ^λ is an arbitrary spacetime vector, so we must have

$$\begin{aligned} 0 &= \partial_\nu\left(h\Theta^a{}_\lambda h^\nu_a\right) + h\Theta^a{}_\mu h^\mu_{a,\lambda} \\ &= h\nabla^\nu\Theta_{\nu\lambda} + h\Theta^{\alpha\nu}K_{\alpha\nu\lambda}. \end{aligned} \tag{18}$$

Given that $K_{(\mu\nu)\rho} = 0$ and using Eq. (16), we get

$$\nabla^\nu\Theta_{\nu\lambda} = 0. \tag{19}$$

Clearly, if $\Theta_{\mu\nu}$ were not symmetric, *i.e.* if the matter action were not invariant under local Lorentz transformations, then $\Theta_{\mu\nu}$ would not be divergence-free either.

We now move to the gravitational sector. As already mentioned, $T^\lambda{}_{\mu\nu}$ behaves like a tensor under spacetime coordinate transformations (the antisymmetry of the last two indices allows us to promote the partial derivatives to covariant ones). The last line of Eq. (6) demonstrates that $K^\rho{}_{\mu\nu}$ is also a spacetime tensor. Consequently, $S^{\rho\mu\nu}$ is also a spacetime tensor and T is a generally covariant scalar. Hence any action constructed with $\mathcal{L}_T$ or $\mathcal{L}$ is generally covariant and invariant under the infinitesimal coordinate transformation $x^\mu \to x^\mu + \epsilon^\mu(x)$.

Some more algebra is needed to check whether such actions are also local Lorentz scalars. From the relation between $\Gamma^\alpha_{\beta\gamma}$ and $\tilde{\Gamma}^\alpha_{\beta\gamma}$ given in Eq. (6), and the fact that the curvature tensor associated with the Weitzenbock connection $\tilde{\Gamma}^\alpha_{\beta\gamma}$ vanishes, we can write the Riemann tensor for the connection $\Gamma^\alpha_{\beta\gamma}$ as [2]

$$\begin{aligned} R^\rho{}_{\mu\lambda\nu} &= \partial_\lambda\Gamma^\rho{}_{\mu\nu} - \partial_\nu\Gamma^\rho{}_{\mu\lambda} + \Gamma^\rho{}_{\sigma\lambda}\Gamma^\sigma{}_{\mu\nu} - \Gamma^\rho{}_{\sigma\nu}\Gamma^\sigma{}_{\mu\lambda} \\ &= \nabla_\nu K^\rho{}_{\mu\lambda} - \nabla_\lambda K^\rho{}_{\mu\nu} + K^\rho{}_{\sigma\nu}K^\sigma{}_{\mu\lambda} - K^\rho{}_{\sigma\lambda}K^\sigma{}_{\mu\nu}. \end{aligned} \tag{20}$$

The corresponding Ricci tensor is then

$$\begin{aligned} R_{\mu\nu} &= \nabla_\nu K^\rho{}_{\mu\rho} - \nabla_\rho K^\rho{}_{\mu\nu} + K^\rho{}_{\sigma\nu}K^\sigma{}_{\mu\rho} - K^\rho{}_{\sigma\rho}K^\sigma{}_\nu \\ &= -\nabla^\rho S_{\nu\rho\mu} - g_{\mu\nu}\nabla^\rho T^\sigma{}_{\rho\sigma} - S^{\rho\sigma}{}_\mu K_{\sigma\rho\nu}, \end{aligned} \tag{21}$$

[1] Dropping this assumption for the matter coupling would lead to Lorentz violations in the matter sector.

and the Ricci scalar

$$R = -T - 2\nabla^\mu \left(T^\nu{}_{\mu\nu}\right). \tag{22}$$

The relations

$$\begin{aligned} K^{(\alpha\beta)\gamma} &= T^{\alpha(\beta\gamma)} = S^{\alpha(\beta\gamma)} = 0, \\ S^\mu{}_{\rho\mu} &= 2K^\mu{}_{\rho\mu} = -2T^\mu{}_{\rho\mu}, \end{aligned} \tag{23}$$

and Eq. (9) were used in deriving Eqs. (21) and (22).

Eq. (22) is very useful, as it shows that T and R differ only by a total divergence. This immediately implies that $\mathcal{L}_T$ is completely equivalent to the Einstein–Hilbert lagrangian density, as the total divergence can be neglected inside an integral, and teleparallel gravity is equivalent to GR. We will see this below at the level of the field equations as well. For the moment, let us focus on a different feature. R is a generally covariant scalar and also a local Lorentz scalar as it can be expressed in terms of the metric and without any reference to the tetrad. Now $\nabla^\mu \left(T^\nu{}_{\mu\nu}\right)$ is also a generally covariant scalar, as $T^\lambda{}_{\mu\nu}$ is a spacetime tensor. Thus, as argued above, T is a generally covariant scalar. However, $\nabla^\mu \left(T^\nu{}_{\mu\nu}\right)$ is *not* a local Lorentz scalar: as one can easily check, it is not invariant under a local Lorentz transformation. Consequently, T is *not* a local Lorentz scalar either.

This has been pointed out already in the literature of standard teleparallel gravity (see Ref. [2] and references therein), *i.e.* when the action considered is constructed simply with $\mathcal{L}_T$, as well as in studies of more general theories where the action is constructed with the Weitzenbock connection and is quadratic in the torsion tensor [18–23]. The former case is very special as the resulting theory is still locally Lorentz invariant. The reason is that the Lorentz breaking term is a total divergence. Therefore, the apparent lack of local Lorentz symmetry at the level of the action appears to be of little importance in teleparallel gravity, *i.e.* when the Lagrangian is just T.

However, the situation is quite different for the $f(T)$ generalisation of teleparallel gravity. It is clear that if T is not a local Lorentz scalar then $f(T)$ cannot be either. Moreover, $f(T)$ cannot be split into two parts with one a local Lorentz scalar and the other a total divergence. This implies that actions of the form given in Eq. (11) are not locally Lorentz invariant. So, $f(T)$ generalizations are not special as the standard teleparallel gravity where $f(T) = T$, but instead behave like the more generic theories where a general action constructed with a Weitzenbock connection is considered. [2]

To get a better understanding of this, we can verify what was said above also at the level of the field equations. Contracting with h^a_ν and using Eqs. (21) and (22), after some algebra we can bring Eq. (12) into the form

$$\begin{aligned} H_{\mu\nu} \equiv{}& f_T G_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \left[f(T) - f_T T\right] + f_{TT} S_{\nu\mu\rho} \nabla^\rho T \\ ={}& 8\pi G \Theta_{\mu\nu}, \end{aligned} \tag{24}$$

where $G_{\mu\nu}$ is the Einstein tensor. When $f(T) = T$, GR is recovered, which verifies the claim that teleparallel gravity and GR are equivalent. In this case the field equations are clearly covariant and the theory is also local Lorentz invariant. In the more general case with $f(T) \neq T$, however, this is not the case. Even though all terms in Eq. (24) are covariant, the last two terms in the first line are not local Lorentz invariant. Hence the field equations are not invariant under a local Lorentz transformation.

Local Lorentz invariance would mean that we can only determine the tetrad up to a local Lorentz transformation; that is, only 10 of the 16 components of the tetrad would be independent and fixing the rest would simply be a gauge choice. Lack of Lorentz invariance implies that the field equations must determine these 6 components as well, leading to a system of 16 equations instead of 10. This is indeed the case: notice that $H_{\mu\nu}$ is not symmetric, but $\Theta_{\mu\nu}$ is, because matter is assumed to couple only to the metric (see above). Therefore, we can split Eq. (24) in the following way

$$H_{(\mu\nu)} = 8\pi G \Theta_{\mu\nu}, \tag{25}$$

$$H_{[\mu\nu]} = 0, \tag{26}$$

which forms a system of 16 component equations. As in GR, we can do away with 4 of these equations by using the usual spacetime gauge symmetry, but there still remain 6 more equations. Note also that since the action and the field equations are covariant, and matter is assumed to couple only to the metric, $H_{\mu\nu}$ does satisfy a generalised contracted Bianchi identity. This means that the zero divergence of $\Theta_{\mu\nu}$ imposes no further constraints. This can be easily argued at the level of the action in analogy with the treatment of $\Theta_{\mu\nu}$ above (modulo the symmetry), but it can also be demonstrated by a direct calculation. Using the definition of $H_{\mu\nu}$ that

$$\nabla^\mu H_{\mu\nu} = f_{TT} \left[R_{\mu\nu} + g_{\mu\nu} \nabla^\sigma T^\rho{}_{\sigma\rho} + \nabla^\sigma S_{\nu\sigma\mu}\right] \nabla^\mu T, \tag{27}$$

and Eq. (21), one gets

$$\begin{aligned} &\nabla^\mu H_{\mu\nu} + f_{TT} \nabla^\mu T\, S^{\rho\sigma}{}_\mu K_{\sigma\rho\nu} \\ &= \nabla^\mu H_{\mu\nu} + H^{\sigma\rho} K_{\sigma\rho\nu} = 0. \end{aligned} \tag{28}$$

For the first equality we have used the fact the $K_{(\sigma\rho)\nu} = 0$. This equation is in direct agreement with the analogous equation for $\Theta_{\mu\nu}$, Eq. (18). If we now use Eq. (26), and $K_{(\sigma\rho)\nu} = 0$ again, then we get

$$\nabla^\mu H_{\mu\nu} = 0. \tag{29}$$

Therefore, on shell, $H_{\mu\nu}$ satisfies a generalized contracted Bianchi identity, as expected from our symmetry analyses above. This is typically the case for covariant theories

[2] Even though this was mentioned already in Ref. [24] for a specific action which falls under the general $f(T)$ class, the implications of the lack of local Lorentz symmetry were not fully spelt out.

with extra degrees of freedom non-minimally coupled to gravity, *e.g.* scalar-tensor gravity theories. Indeed, the theory appears to propagate more degrees of freedom, as is consistent with the lack of symmetry. Eqs. (25) and (26) are second-order differential equations but they are expected to harbour more degrees of freedom than the two graviton polarizations of GR, contrary to what has been implied in the literature. Also, the fact that the field equations are second order does not mean that extra excitations will necessarily be healthy. For instance, a wrong sign could lead to ghosts or classically unstable modes. The dynamics of the extra degrees of freedom of $f(T)$ gravity certainly deserves further investigation.

Lack of local Lorentz symmetry implies that there is no freedom to fix any of the components of the tetrad. They must all be determined by the field equations. Now, suppose that we want to impose a metric ansatz based on specific spacetime symmetry assumptions. Does this imply a certain ansatz for the tetrad? The answer is, only partially. Eq. (1) provides only 10 algebraic relations between the 10 independent metric components and the 16 independent tetrad components. Were the theory local Lorentz invariant, one would be able to fix the remaining 6 tetrad components. In absence of the symmetry this is not an option and they need to be determined by the field equations.

For instance, assuming a spatially flat Friedmann–Lemaître–Robertson–Walker line element,

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2) \tag{30}$$

does not uniquely lead to the tetrad choice

$$h^a_\mu = \text{diag}(1, a(t), a(t), a(t)), \tag{31}$$

as is very commonly assumed in the literature. There is simply not enough freedom to make this assumption and one would need to resort to the field equations and explicitly show, not only the consistency, but also the uniqueness of this specific choice.

To summarise, we have studied the symmetries and the dynamics of $f(T)$ theories of gravity. We have shown that, even though they are covariant, such theories are not local Lorentz invariant, with the exception of the $f(T) = T$ case, which have been extensively studied in the literature and is equivalent to GR. This fact has several consequences. First, it is expected to lead to strong preferred-frame effects which should in turn be crucial for the viability of the theory. This casts serious doubt on whether such theories can provide interesting alternatives to GR. Note that even though matter will not 'feel' the preferred frame effects because it is only coupled to the metric, these effects still leave an observational signature in gravitational experiments, as in the case of Einstein-aether theory [25]. Another consequence is that the lack of symmetry implies the presence of more degrees of freedom. Indeed, there appear to be 6 more dynamical equations than in GR. Even though all equations are second order in derivatives, this is not enough to guarantee that the extra excitations will be well behaved. The lack of Lorentz symmetry also presents a serious computational complication because there is no freedom to gauge fix tetrad components.

We hope that this analysis will prompt a search for a deeper understanding of the dynamics of $f(T)$ gravity, the presence of extra degrees of freedom in these theories, and their cosmological behaviour. There also needs to be a thorough study on the observational consequences of local Lorentz symmetry violations. We hope to address these issues in future work.

Before closing let us point out that it is rather trivial to modify $f(T)$ theory in order to make it manifestly Lorentz invariant. If the partial derivative is replace by a Lorentz covariant derivative (see Ref. [17]) in the definition of $T^\lambda{}_{\mu\nu}$, Eq. (5), and then one defines a quantity $\bar{T}$ in the same way as T is defined here, $\bar{T}$ or $f(\bar{T})$ will be manifestly locally Lorentz invariant, see also Ref. [26]. Note, however, that even though such a theory will reduce to $f(T)$ gravity in some local Lorentz frames (those for which the Lorentz covariant derivative becomes a partial derivative), it will generically have different dynamics. It is, therefore, a different theory, which might deserve further investigation.

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